0.1 Basic Courses

101. Calculus I

- Real Numbers. Axiom of completeness and consequences.
- Convergence of sequences.
- Functions. Algebraic functions, preliminary definition of the trigonometric functions, exponential function.
- Limits and Continuity. Intermediate value theorem. Continuous functions on closed intervals. Monotone functions. Inverse functions. Logarithmic function.
- Derivative. Differentiation rules. Mean Value Theorems. Significance of the derivative.
- Supplements: Countable and uncountable sets. Construction of the real numbers system (Dedekind cuts).

121. Linear Algebra I

- Matrices and systems of linear equations.
- Vector spaces.
- Linear mappings.
- Matrices and linear mappings.
- Determinants.
- Systems of linear equations.

122. Analytic Geometry

- Vector Calculus and applications.
- Analytic Geometry in the plane.
- Elements of Analytic Geometry in three–dimensional space.
- Euclidean Geometry in $\mathbb{R}^n$.

141. Computer Science I

Algorithmic resulution of problems and programming in MATLAB

- Basic concepts of computers and algorithms.
- Variables, expressions, asignments, input/output.
- Commands of control and repetition.
- Functions, recursion.
- Matrices/order and basic data structures.

- Searching/ordering, efficiency of algorithms.
- Computational representation of numbers.
- Mathematical applications, simulation/modeling, graphics.
- Introduction to LaTeX.

201. Calculus II

- Subsequences, Bolzano–Weierstrass theorem. Cluster points, limit superior and limit inferior. Cauchy sequences.
- Convergence of series.
- Uniform continuity.
- Convex and concave functions. Jensen's inequality and applications.
- Riemann integral. Definition and existence theorems. Linearity and order properties. Mean value theorems.
- Fundamental Theorem of Calculus.
- Techniques of integration.
- Taylor's theorem. Power series. Interchange of limit operations.
- Supplement: Definition of the trigonometric functions. Alternative definitions of the logarithmic and the exponential function.

221. Linear Algebra II

Objectives: Similar matrices are known to represent a given linear endomorphism with respect to different choices of a basis. A fundamental question is whether, given such an endomorphism f of a finite–dimensional vector space V, there exists a basis of V with respect to which the matrix of f has a specific ”simple form”.

The goal of this course is the study of some of these simple forms, such as diagonal, triangular or Jordan canonical form. To achieve this goal we will study concepts (among others) such as that of an eigenvalue, eigenvector, characteristic polynomial and minimal polynomial.

- Determinants and Polynomials.
- Eigenvalues and Eigenvectors.
- Diagonalizable Linear Transformations.
- Triangulable Linear Transformations and the Cayley–Hamilton Theorem.
- Minimal Polynomial.
- Criteria for Diagonalizability.
- Primary Decomposition, Jordan Canonical Form.
- The Standard Inner Product.

- Unitary and Hermitian Matrices, Diagonalization of Hermitian Matrices.
- Quadratic Forms.

241. Probability I

- Sample space and events. Axiomatic foundation of Probability. Finite sample spaces and classical Probability. Conditional Probability and stochastic independence.
- Random variable and distribution function. Discrete and continuous random variables. Distribution of a function of a random variable. Moments of random variables, mean value and variance. Chebyshev's inequality.
- Univariate discrete distributions, especially: Bernoulli and Binomial distribution, Geometric and Pascal distribution, Poisson distribution.
- Univariate continuous distributions, especially: continuous Uniform distribution, Exponential and Gamma distribution, Beta distribution, Normal distribution.
- Bivariate random variable and distribution function. Discrete and continuous bivariate random variables.
- Conditional probabilities and independent random variables.
- Generating functions of probabilities and moments. Laws of large numbers of Bernoulli and Chebyshev. Central limit theorem of Lindeberg–Levy and applications.

301. Calculus III

- Vector Calculus in three–dimensional Euclidean space: vectors, scalar product and cross product, applications.
- Analytic Geometry in three–dimensional Euclidean space: planes, curves, surfaces, cylindrical and spherical coordinates.
- Linear Algebra of Euclidean space: algebraic structure, matrices and linear mappings.
- Topology of Euclidean space: sequences, open and closed sets, bounded and compact sets, connected sets, boundary.
- Functions of several variables: limits and continuity, fundamental theorems for continuous functions, uniform continuity.
- Differentiability of functions of several variables: partial derivatives, gradient, differential, tangent plane, linearization and approximation, main theorems of Differential Calculus, chain rule, inverse and implicit function theorem, maxima and minima, applications.
- Double and Triple Integral: definition and properties, area and volume computations, techniques of integration, change of variables, polar, cylindrical and spherical transformation.
- Line integrals: parametrization and parametrized curves, length of a parametrized curve, definition and properties of line integrals, computation, path independence conditions, applications.

- Surface integrals: parametric surfaces, surface area, definition and properties of surface integrals, computations, applications.
- Vector Analysis: differential operators of scalar and vector fields, theorems of Green and Stokes, divergence theorem, applications.

302. Differential Equations I

- First order differential equations (linear, Bernoulli, Riccati, equations with separable variables, homogeneous equations, exact equations, Euler multipliers).
- Existence, uniqueness and continuation of solutions, well–posed problems.
- Second order linear differential equations: general theory of homogeneous and non–homogeneous differential equations.
- Sturm's separation and comparison theorems.
- Power series solutions.
- Systems of first–order linear differential equations: general theory for homogeneous and non–homogeneous systems.
- Sturm–Liouville boundary value problems.
- Laplace transform.
- Introduction to the qualitative theory of ordinary differential equations.

401. Real Analysis

- Countable and uncountable sets.
- Metric spaces: definitions, basic properties and examples, topological notions, equivalent metrics, bounded and totally bounded sets.
- Continuity of functions on metric spaces: definitions, properties of continuous functions, isometries, Lipschitz functions, uniform continuity.
- Completeness: complete metric spaces (definition, basic properties, examples), fixed point theorems and applications to differential equations, Cantor theorem, Baire theorem and applications.
- Compactness: definition (via coverings) and basic properties. Compactness and Continuity. Characterizations of compactness. Finite products of compact metric spaces.
- Separability.
- Cantor set.
- Sequences and series of functions: pointwise and uniform convergence (definition, basic properties and examples). Weierstrass test for series of functions. Uniform convergence and continuity, integration, differentiation.
- Continuous functions on compact metric spaces: structure of C(X). Weierstrass approximation theorem.

421. Basic Algebra

- Basic number theory: divisidility of integers, greatest common divisor, congruences modulo n.
- Basic ring theory: rings, integral domains and fields, polynomials over a field (divisibility, greatest common divisor, roots), homomorphisms, ideals and quotient rings.
- Basic group theory: symmetries, permutations, groups and subgroups, Lagrange's theorem, cyclic groups, homomorphisms, normal subgroups and quotient groups.

541. Mathematical Statistics

- Descriptive Statistics.
- Distribution Families.
- Exponential Distribution Families.
- Sufficiency and Completeness.
- Minimum Variance Unbiased Estimators.
- Cramer–Rao Inequality
- Efficient Estimators
- Consistent Estimators
- Maximum Likelihood Estimators and Moment Estimators
- Bayes and Minimax Estimators
- Confidence Intervals
- Statistical Hypothesis Testing

634. Differential Geometry of Curves and Surfaces

- Regular curves, arc length, parametrization with respect to arc length, curvature and torsion, Frenet–Serret frame, fundamental theorem of curves.
- Regular surfaces, tangent plane, Gauss map and shape operator, second fundamental form, principal curvatures, Gauss curvature and mean curvature, isometries, Gauss's Theorem Egregium, intrinsic geometry, geodesics, Gauss Bonnet theorem.

701. Complex Analysis I

- Complex numbers.
- Topology of metric spaces.
- Holomorphic functions, Cauchy–Riemann equations.
- Power series, Taylor's theorem, complex integration.

- Cauchy Integral Theorem, maximum principle, Morera theorem, Liouville theorem, Fundamental Theorem of Algebra.
- Analytic continuation, sequences of holomorphic functions.
- Poles and roots. Laurent series, residue theorem, applications to the computation of improper integrals.

0.2 Pure Mathematics I

411. Partial Differential Equations

- Integral curves and surfaces of vector fields.
- Quasilinear partial differential equations of first order. The initial value problem. The initial value problem for conservation laws. Shock waves.
- Classification and canonical forms of second order partial differential equations.
- Elliptic equations: boundary value problems, the method of separation of variables, eigenex–pansions in cartesian, polar and cylindrical coordinates, fundamental solutions, integral representations, Poisson integral, Green functions, basic properties of harmonic functions.
- Parabolic equations: initial–boundary value problems, the non–homogeneous problem, fundamental solutions, integral representations, Fourier transform.
- Hyperbolic equations: initial–boundary value problems, the non–homogeneous problem, fundamental solutions, Fourier transform.

423. Rings and Modules

- Basic notions on modules.
- Factorization in integral domains (Euclidean rings, principal ideal domains, unique factorization domains).
- Free modules.
- Structure theory for finitely generated modules over principal ideal domains.
- Applications to finitely generated abelian groups and normal forms of matrices.

511. Measure Theory

- Measure spaces, outer measures, Lebesgue measure.
- Measurable functions.
- Lebesgue integral, comparison with the Riemann integral.
- Sequences of measurable functions, L_p spaces.
- Product measures, Fubini theorem.

- Signed measures, Radon–Nikodym theorem.

513. Mathematical Logic

- Propositional Calculus.
- First Order Predicate Calculus.
- Completeness Theorem and Compactness Theorem for First Order Predicate Calculus.
- Lowenheim–Skolem Theorems.
- Elements of Model Theory.

532. Number Theory

- Prime Numbers and the fundamental theorem of Arithmetic
- Divisibility, GCD, LCM, Euclid’s algorithm.
- Linear Diophantine, equations, Pythagorian triples.
- Arithmetic functions, Euler function and inversion formula.
- Congruences, Chinese remainder theorem.
- Recursive solution of polynomial equtions modulo prime powers.
- Introduction to cryptography and the RSA algorithm.
- Primitive roots, indices, and Fermat’s little theorem
- Quadratic residues, quadratic reciprocity law, and computations with Legendre and Jacobi symbols.

533. Foundations of Geometry: An Introduction

- Axiomatic foundations of Geometry, Hilbert□s axiomatic system.
- The Absolute Geometry.
- Topics of the plane Hyperbolic Geometry.
- Klein□s Erlagen Program, Geometric Transformations for the Plane and Space Geometry.
- Euclidean and non–Euclidean Geometries.

602. Introduction to Functional Analysis

- Preliminaries: elementary facts on vector spaces and metric spaces.
- Banach spaces: basic notions and examples (classical sequence spaces).
- Properties of Banach spaces, finite dimensional spaces (equivalence of norms, compactness and finite dimension).

- Hilbert spaces (basic notions and examples, properties of Hilbert spaces, orthogonality, orthonormal families, bases).
- Linear operators (bounded linear operators, linear operators on Banach spaces, the dual space of a Banach space, the dual space of a Hilbert space, bounded linear operators on Hilbert spaces).
- Fundamental principles of Banach space theory: Hahn–Banach theorem, uniform boundedness principle, open mapping theorem, closed graph theorem. Reflexivity and separability (reflexive Banach spaces, reflexivity of Hilbert spaces, separable Banach and Hilbert spaces).
- Weak and weak* convergence: weak convergence and weak* convergence of sequences in Banach and Hilbert spaces, bounded and weakly bounded sets in Banach and Hilbert spaces.

605. Fourier Analysis and Lebesgue Integral

- Trigonometric polynomials, trigonometric series.
- Orthogonal systems of functions, examples.
- Fourier series of a function. Bessel inequality.
- Dirichlet and Fejer kernels. Fejer theorem. Parseval theorem.
- Examples: Fourier series expansions of periodic functions
- Construction of Lebesgue measure.
- Measure spaces. Measurable functions
- Simple functions. Integration.
- Comparison of the Lebesgue integral with the Riemann integral.

714. Introduction to Topology

- Topological spaces: topology, topological space, main topological notions, base, sub-base, local base, subspaces of a topological space.
- Continuous functions on topological spaces. Product topology, metric topologies.
- Convergence: nets and subnets, convergence of sequences, convergence of nets, continuity of functions through nets.
- Compactness: compact topological spaces, basic properties, continuity and compactness, compact metric spaces.
- Connectedness: connected topological spaces and basic properties, connected components, continuity and connectedness.
- Axioms of countability and axioms of separation. Urysohn lemma, Urysohn mertrization theorem, Tychonoff product theorem.
- Topologies of function spaces (pointwise topology, compact–open topology).

721. Introduction to Differential Manifolds

- Differential manifolds, the topology of manifolds, examples. Morphisms. Exercises.
- Tangent space, derivations, point derivation. Tangent bundle, derivative map. Examples, exercises.
- Vector fields, Lie product, invariant vector fields, integral curves of vector fields, differentiable flows. Examples, exercises.
- Lie groups. Lie algebra of a Lie group. Exponential map of a Lie group. Normal coordinates. Examples, exercises.

821. Galois Theory

- Rings and characteristic, field of quotients, maximal and prime ideals and their quotients.
- Polynomial rings of one variable and their ideals, division algorithm. Irreducible polynomials in Z, Q and Gauss lemma. Irreducibility criteria.
- Fields and their extension, algebraic numbers. Construction with compass and ruler.
- Galois group of an extension, splitting field. Finite extensions of groups and isomorphisms. Fundamental theorem of Galois theory.
- Finite fields and their extensions, cyclotomic polynomials
- Solvable groups, criterion of solvability of equations, the general equation of degree > 4 cannot be solved with radicals.
- Simple extensions and characteristic
- Applications: Solving equations of degree < 5 with radicals, resolvent. General polynomial of degree n, regular polygons, fundamental theorem of algebra.

834. Group Theory

- Definition and basic properites of groups, order, subgroups. Lagrange's theorem, normal subgroup.
- Expression of groups with generators and relations, free groups.
- Cyclic groups, dihedral groups, symmetric group. Computations in the symmetric group. Conjugate groups, conjugation classes of elements, conjugation classes in the symmetric group.
- Isomorphism theorems, the theorem of Caley.
- Quotient groups, products of groups, extensions of groups.
- Classification of finitely generated abelian groups.
- Action of groups on Sets, the theorem of Cauchy.
- Sylow theorems, classification of groups ≤9.
- Normal series, solvable and nilpotent groups.

0.3 Applied Mathematics I

151. Combinatorics

- Basic counting principles, sums and products, recursive relations.
- Permutations, combinations, divisions and partitions of a finite set, integer solutions of linear equations.
- Binomial and multinomial coefficients. Computations of finite sums.
- The principle of inclusion–exclusion.
- Univariate generating functions. Generating functions for combinations and permutations.
- Distributions and occupancy.

251. Computer Science II
JAVA object–oriented programming language. Specific topics:

- Review of basic topics (syntax, primitive data types, expressions, operators, flow control, logical operations, comparisons, type casting).
- Classes, method types, constructors, objects.
- Packages.
- Inheritance and other object–oriented programming principles.
- Static data structures (arrays).
- Dynamic data structures (vectors).
- Exceptions handling.
- Recursion.
- Window environments.
- Applets.

252. Discrete Mathematics
Basic topics

- Basic principles of enumeration and applications (enumeration of sets, words, permutations).
- Binomial coefficients and their properties.
- Ordinary and exponential generating functions. Applications in enumerating sets, permutations, partitions of integers/sets.
- Special numbers (Stirling, Bell, Catalan).
- The principle of inclusion–exclusion.
- Recurrence relations and difference equations.

- Computation of sums.
- The pigeonhole principle.
- Applications in problems of discrete probabilities and enumeration of graphs (e.g., Cayley's formula for the number of trees, enumeration of matchings and colorings, Euler's formula for planar graphs).

Time permitting the following topics will also be covered:

- Elements of graph theory.
- Elements of extremal combinatorics.
- Elements of discrete geometry.
- Polya's theory.
- Elements of analytic combinatorics.

341. Numerical Analysis I

- Computer arithmetic and round–off errors.
- Solutions of equations in one variable (the bisection method, fixed–point iteration, the Newton–Raphson method).
- Direct methods for solving linear systems (Gaussian elimination, norms of vectors and matrices, condition number).
- Polynomial interpolation and splines.
- Numerical integration (trapezoidal and Simpson□s' rules, Newton–Cotes formulas).

342. Operations Research: Mathematical Programming

- Linear Programming: Introduction, Examples of formulations.
- The Simplex method and its variations.
- Duality theory and applications.
- Optimality equations for finite and infinite horizon problems.
- Applications to problems in network flows, inventory management, maintenance and replacement of equipment.

411. Partial Differential Equations

- Integral curves and surfaces of vector fields.
- Quasilinear partial differential equations of first order. The initial value problem. The initial value problem for conservation laws. Shock waves.
- Classification and canonical forms of second order partial differential equations.

- Elliptic equations: boundary value problems, the method of separation of variables, eigenexpansions in cartesian, polar and cylindrical coordinates, fundamental solutions, integral representations, Poisson integral, Green functions, basic properties of harmonic functions.
- Parabolic equations: initial–boundary value problems, the non–homogeneous problem, fundamental solutions, integral representations, Fourier transform.
- Hyperbolic equations: initial–boundary value problems, the non–homogeneous problem, fundamental solutions, Fourier transform.

442. Probability II

- Σ–algebras.
- Measures.
- Measurable functions.
- The Lebesgue integral.
- Modes of convergence for random variables, independence.
- The Borel–Cantelli lemmas.
- Kolmogorov's 0–1 law.
- The strong law of large numbers.
- Characteristic functions.
- Convergence in distribution.
- The central limit theorem.
- Large deviations and Cramer's theorem.

552. Operations Research: Stochastic Models

- Stochastic systems and stochastic processes.
- Intoduction to queueing systems.
- Birth–death processes.
- Renewal theory and applications.
- Continuous–time Markov chains.
- Applications of discrete and continuous time Markov chains in queueing and inventory control.

605. Fourier Analysis and Lebesgue Integral

- Trigonometric polynomials, trigonometric series.
- Orthogonal systems of functions, examples.

- Fourier series of a function. Bessel inequality.
- Dirichlet and Fejer kernels. Fejer theorem. Parseval theorem.
- Examples: Fourier series expansions of periodic functions
- Construction of Lebesgue measure.
- Measure spaces. Measurable functions
- Simple functions. Integration.
- Comparison of the Lebesgue integral with the Riemann integral.

651. Stochastic Processes

- Distribution of a stochastic process.
- Stationarity.
- Discrete time Markov chains (transition probabilities, two–state chains, classification of states, stationary distribution).
- Continuous time Markov chains (Poisson process, interarrival times and waiting times, birth–death processes, linear birth–death process, Furry–Yule process, death process, applications).

654. Linear Models

- Hypothesis Tests
- Nonparametric Inference
- Linear Models
- Analysis of Variance

0.4 Pure Mathematics II

Mathematical Analysis

110. Foundations of Mathematics

- Sets – Relations – Functions.
- Propositional Calculus.
- Natural numbers: Peano axioms, induction, well–ordering principle.
- Real numbers. Cardinality, countable and uncountable sets.
- Complex numbers – polynomials – Gauss elimination.

432. Matrix Analysis and Applications

- Representations of linear and multilinear functions.
- Basic classes of matrices and their important properties.
- Matrix norms and condition number.
- Singular value decomposition and its applications.
- Sensitivity and stability of linear systems.
- Fundamental subspaces defined by a matrix.
- Invariant subspaces, pseudoinverses and least squares approximation.
- Hermitian, symmetric positive–definite and non–negative matrices.
- Eigenvalue problems, minimax principle for eigenvalues, bounds for eigenvalues and perturbation theory.
- Generalized eigenvalues–eigenvectors problem.
- Polynomial matrices and applications (Smith canonical form, Smith–MacMillan form and Hermitian form).
- Linear matrix equations, generalized inverses.
- Functions of matrices. Difference equations.
- Exponential map and applications to differential equations.
- Stability of differential equations.

514. Convex Analysis

- Convex Sets. Convex and concave functions.
- Theorems of Caratheodory, Helly, Radon. Applications to combinatorial geometry and approximation theory.
- Metric projection. Supporting planes. Separation theorems. Duality. Support function.
- Extreme and exposed points. Theorem of Minkowski–Krein–Milman and applications (Birkhoff's polytope)
- Hausdorff metric. Blaschke selection theorem. Steiner symmetrization and geometric applications.
- Volume in n–dimensional Euclidean space. Volume and high dimension.
- Brunn–Minkowski inequality. Isoperimetric problems.
- Topics: geometric inequalities, geometry of numbers, finite dimensional normed spaces, ellipsoids and algorithmic volume computation, geometric probabilities.

518. Algorithm Design and Analysis

- The notion of algorithm: run–time computation, proofs of correctness, recurrences, worst–case performance, average case performance.
- General methods of algorithmic design: divide and conquer method, dynamic programming, greedy algorithms.
- Graphs and graph algorithms: representation of graphs, graph transversals, minimum–cost spanning trees, shortest paths.
- Algorithms for network problems: network flows, augmenting paths, matchings in bipartite graphs, minimum cost flows.
- Topics: pattern matching, data compression, public key cryptography, approximate algorithms.

611. Set Theory

- Intuitive set theory.
- Zermelo–Fraenkel axioms for set theory.
- Ordinal numbers, cardinal numbers.
- Axiom of choice and equivalents.
- Subsets of the real number system, continuum hypothesis, generalized continuum hypothesis.
- Constructible sets.

614. Recursive Functions

- The notion of computability.
- Primitive recursive functions.
- Recursive functions.
- Church Thesis.
- Godel's enumeration of the syntax of a first order language.
- Representability.
- Incompleteness Theorem.

615. Geometric Analysis

- Inverse and implicit function theorems, surfaces in Rn, Sard theorem, partitions of unity.
- Change of variables formula in multiple integrals, differential forms in $\mathbb{R}^n$ and on surfaces, Poincare lemma, ∂–equation.
- Stokes theorem, area element, Gauss divergence theorem, degree theory, examples of de Rham cohomology. Applications.

616. Approximation Theory

- Uniform approximation. Weierstrass approximation theorem.
- Best approximation in normed linear spaces.
- Polynomial interpolation (Lagrange–Newton), interpolation with piecewise polynomial functions (splines).
- Least squares approximation.
- Orthogonal polynomials.
- Numerical integration based on interpolation (Newton–Cotes), Gaussian quadrature, Romberg integration.

618. Computational Complexity

- Models of computability, Turing machines, the notion of complexity of a problem. Complexity classes: Class PSPACE, Savitch’s theorem, Classes P and EXP.
- Non–Deterministic Turing machines.
- Classes NP and co–NP. The Projection Theorem. Reducibilities and Completeness, NP–hardness.
- Coo$k$-Levin theorem, NP–complete problems, Methods of proof of NP–completeness, Pseudo-polynomiality, Strongly NP–complete problems.
- NP–completness and approximability, EXP–complete and PSPACE–complete problems.

712. Linear Operators

- Euclidean spaces, inner (scalar) products on infinite–dimensional spaces. Completeness, Hilbert space: basic properties.
- Bounded operators: examples. The adjoint of an operator, classes of operators, orthogonal projections.
- Finite rank operators, compact operators, integral operators.
- Diagonalizing operators: the Spectral Theorem for compact normal operators. Applications.
- Complements: Compact operators on Banach spaces: Riesz–Schauder theory. Invariant subspaces of compact operators.

Prerequisites: Infinitesimal calculus, Linear Algebra, Real Analysis (metric spaces). No knowledge of Functional Analysis is presupposed.

813. Complex Analysis II

- Analytic functions. Cauchy integral and applications.
- Harmonic functions.

- Conformal mapping.
- Mittag–Leffler expansions.
- Weierstrass factorization theorem.
- Periodic functions.
- Special functions.

814. Control Theory

- Mathematical models of physical systems.
- Linearization and transfer functions, state–space approach to linear systems theory.
- Segre–Weyr method for the Jordan form of a linear operator.
- Functions of square matrices. Functions $I(t)$, $\delta(t)$, Laplace transform.
- General solution of time dependent linear dynamical systems.
- Root locus method.
- Controllability and Observability.
- Realisation theory. Feedback.
- Stability (general theory). Liapunov theorems.
- Stability criteria for linear dynamical systems.

815. Optimization

- Convex sets, supporting and separating planes.
- Separation theorems.
- Extreme points, Minkowski theorem.
- Polyhedra, characterization of the extreme points of polyhedra.
- Applications to Linear Programming.
- Convex functions, differentiability, maxima and minima.
- Optimization under equality and inequality constraints.
- Lagrange multiplier functions. Karush–Kuch–Tucker conditions.
- Duality, Lagrange duality and saddle points.
- Applications to Economic theory.

817. Applied Fourier Analysis

- Basic facts from the theory of Fourier series.

- Fourier Transform, Discrete Fourier Transform, Fast Fourier Transform.
- Applications to differential equations and signal processing.

711. Topics in Mathematical Analysis I

The aim of this course is the in-depth study of some topic which could belong to any field of Mathematical Analysis and which may be decided following a discussion with the students. An important aspect of the course is the active participation of students by means of presentations. Indacatively, such topics could be: Analytic Number Theory, Infinite Combinatorics, Harmonic Analysis, Harmonic Analysis on locally compact Abelian Groups, General Topology, Geometric Measure Theory, Ergodic Theory, Topics on the History of Mathematical Analysis, Measure Theory, Real Analysis, Set Theory, Operator Theory, Calculus of Variations, Partial Differential Equations, Complex Analysis, Probabilistic Methods in Combinatorics, Integral Tranforms, Functional Analysis, Spectral Theory and others.

812. Topics in Mathematical Analysis II

The aim of this course is the in-depth study of some topic which could belong to any field of Mathematical Analysis and which may be decided following a discussion with the students. An important aspect of the course is the active participation of students by means of presentations. Indacatively, such topics could be: Analytic Number Theory, Infinite Combinatorics, Harmonic Analysis, Harmonic Analysis on locally compact Abelian Groups, General Topology, Geometric Measure Theory, Ergodic Theory, Topics on the History of Mathematical Analysis, Measure Theory, Real Analysis, Set Theory, Operator Theory, Calculus of Variations, Partial Differential Equations, Complex Analysis, Probabilistic Methods in Combinatorics, Integral Tranforms, Functional Analysis, Spectral Theory and others.

Algebra and Geometry
431. Projective Geometry

- Axioms of the affine plane and of the projective plane.
- Principle of duality.
- Completion of an affine plane. Deletion of a projective plane.
- Morphisms of projective planes and collineations.
- Groups of homologies and of elations.
- The projective plane $P_2(\mathbb{R})$.
- Classification of the homologies and of the relations of $P_2(\mathbb{R})$.
- Pappian and Desarguesian projective planes.
- Pascal's and Briancon's theorems.
- The division ring of a Desarguesian projective plane.

439. Computational Algebra

- Multivariable polynomial rings

- Multivariable polynomial systems
- Groebner basis, Hilbert's basis theorem
- Properties of Groebner bases, and solution of polynomial systems
- Basic principles of robotics
- Software packages

534. Commutative Algebra and Applications

- Basics: Ideals, quotient rings, the radical, prime and maximal ideals
- Modules,.
- chain conditions, Noetherian and Artinian rings
- Hilbert's basis theorem
- Integral dependence, integral extensions, algebraic integers and Noether normalization.
- Nullstellensatz and applications to Geometry.
- Localization and primary decomposition.
- Discrete valuation rings.

639. Finite Fields and Coding Theory

- Finite Fields: Definition, existence of finite fields of given order p^h. Subfields of finite fields,

  Υποσώματα πεπερασμένων σωμάτων, primitive elements of a finite field, finite extensions of finite fields. Polynomials over finite fields, irreducible polynomias, the field of roots of a given polynomial over a finite field, the minimal polynomial of an element in an extension of a finite field. Roots of unity, factorization of x^n-1, principal roots of unity. Cyclotomic polynomials over finite fields. Automorphisms of finite fields.
- Codes:

  Elements of coding theory, transmition errors, error detection and correction. The maximum propability principle for decoding. The least distance principle. Families of codes, linear and non-linear codes. Matrix generators and parity detection matrices. The dual code of a linear code. Linear coding and decoding syndrome. Cyclic codes, polynomial generators and control polynomial of a linear code. Cyclic codes and roots of unity. Encoding and decoding in cyclic codes
- Applications: Binary cyclic codes, Hamming codes, Reed-Muller, Golay and Reed-Solomon codes, quadratic residue codes. BCH and codes of maximal (minimal) distance.

734. Algebraic Combinatorics

Algebraic Combinatorics is the area of mathematics which either uses tools from algebra or related fields of theoretical mathematics to solve purely combinatorial problems, or uses combinatorial methods to solve problems that come up in these fields. The objective of this course is to demonstrate this interaction through specific examples of problems, assuming only a minimum prerequisite on algebra and combinatorics. More specifically, the objective is to familiarize students in theoretical mathematics with combinatorial methods and their importance in pure mathematics students in applied mathematics with ways in which tools from pure mathematics (e.g. linear algebra) can be used to solve practical combinatorial problems.

- Review of fundamental principles and techniques of enumeration, with an emphasis on bijective proofs and the method of generating functions. Examples (sets, permutations, integer partitions etc) (2 weeks).
- Permutations as words, elements of the symmetric group, disjoint union of cycles (cycle structure), 0–1 matrices, increasing trees etc. Permutation enumeration (inversions, cycles, descents, excedences, fixed points, alternating permutations, major index and MacMahon's Theorem). Permutations of multisets, inversions and q–binomial coefficients. Young tableaux and the hoo$k$-length formula, the Robinson–Schensted correspondence, Knuth equivalence, Schutzenberger's teasing game, applications to monotone subsequences, the evacuation tableau and Schutzenberger's theorem on the inverse and reverse permutation. The weak Bruhat order and applications on the enumeration of reduced decompositions of permutations (7 weeks).
- Elements of algebraic graph theory, the adjacency matrix of a graph (directed or not), eigenvalues and enumeration of walks. The Laplacian matrix, spanning trees and the Matrix–Tree Theorem, applications to complete (Cayley's formula) and bipartite graphs. Walks in the Young lattice and differential partially ordered sets. Applications of linear algebra on topics such as: the unimodality of q–binomial coefficients, existence of matchings in graphs and Sperner's Theorem and its generalizations (4 weeks).

736. Homological Algebra and Categories

- Elements of category theory.
- Free, projective and injective modules.
- Homology, Ext, Tor.
- Applications.

831. Differential Forms

- Multilinear mappings. Symmetric and antisymmetric multilinear mappings
- Tensor products of linear spaces and mappings
- Duality. Covariant and contravariant tensors.
- Tensor algebras.
- Tangent and cotangent bundle of a differentialbe manifold.
- Basic vector fields and 1-forms.
- Differential forms of order k.

- Poincaré's Lemma.
- Exactness of the de Rham complex.
- Integration of differential forms.
- Stokes' Theorem.

832. Algebraic Topology

- Path connected spaces, homotopy of paths.
- Fundamental group.
- Group actions on topological spaces.
- Covering spaces, fundamental group of the circle (Brouwer's fixed point theorem, fundamental theorem of Algebra).
- Classification of covering spaces, Borsu$k$-Ulam theorem.
- Elements of singular homology theory.

870. Mathematical Physics

- Introduction to differential geometry.
- Langrangian mechanics and the tangent bundle.
- Symmetries and Noether's theorem.
- Legendre transformation.
- Hamiltonian mechanics and the cotangent bundle.
- The canonical symplectic form and Liouville's theorem.
- Poisson bracket, Poincare's theorem and the Hamilton–Jacobi equation.
- Introduction to Symplectic and Poisson geometry.

732. Topics in Algebra and Geometry I

The aim of this course is the in-depth study of some topic which could belong to any field of Algebra and Geometry and which may be decided following a discussion with the students. An important aspect of the course is the active participation of students by means of presentations.

Indicatively, such topics could be: algebraic number theory, algebraic topology, commutative algebra, combinatorics, algebraic geometry, Galois theory, invariant theory, group theory, representation theory, differential geometry, Lie algebras etc.

833. Topics in Algebra and Geometry II

The aim of this course is the in-depth study of some topic which could belong to any field of Algebra and Geometry and which may be decided following a discussion with the students. An important aspect of the course is the active participation of students by means of presentations.

Indicatively, such topics could be: algebraic number theory, algebraic topology, commutative algebra, combinatorics, algebraic geometry, Galois theory, invariant theory, group theory, representation theory, differential geometry, Lie algebras etc.

0.5 Applied Mathematics II

Applied Mathematics

352. Data Structures

- Introduction: the concept of Abstract Data Type.
- Matrices, records, sets, strings.
- Stacks, queues, lists, trees (binary search trees).
- Data structures for Graphs.

373. Graph Theory

- Isomorhpisms, Automorphisms, Group of Automorphisms.
- Transformations and relations on graphs.
- Degrees, Density, min-max theorem on degeneracy.
- Paths, Cycles, Diameter, Radius, Center, Girth, Perimeter.
- Connectivity, biconnectivity, Menger's theorem.
- Trees, forests, spanning trees.
- Planar graphs, Duality, density and planarity, Theorem of Kuratowski.
- Graph Coloring, Bipartitite Graphs, Coloring and degeneracy, Theorem of Heawood.
- Cliques, Independent sets, Ramsey Numbers.
- Coverings and Matchings, Theorem of Hall, Perfect Matchings, Theorem of Tutte.
- Euler cycles, Hamilton cycles.
- Elements of Structural Graph Theory.

432. Matrix Analysis and Applications

- Representations of linear and multilinear functions.
- Basic classes of matrices and their important properties.
- Matrix norms and condition number.
- Singular value decomposition and its applications.
- Sensitivity and stability of linear systems.
- Fundamental subspaces defined by a matrix.
- Invariant subspaces, pseudoinverses and least squares approximation.
- Hermitian, symmetric positive–definite and non–negative matrices.

- Eigenvalue problems, minimax principle for eigenvalues, bounds for eigenvalues and perturbation theory.
- Generalized eigenvalues–eigenvectors problem.
- Polynomial matrices and applications (Smith canonical form, Smith–MacMillan form and Hermitian form).
- Linear matrix equations, generalized inverses.
- Functions of matrices. Difference equations.
- Exponential map and applications to differential equations.
- Stability of differential equations.

453. Computer Graphics

- Output Primitives: Pixels, Points and Lines.Line–Drawing Algorithms. Bresenham's Line Algorithm. Circle–Generating Algorithms. Bresenham's Circle Algorithm. Ellipses.
- Two–Dimensional Transformations: Basic Transformations: Translation, Scaling, Rotation. Matrix Representations and Homogeneous Coordinates. Composite Transformations. Scaling Relative to a fixed Point, Rotation about a Pivot Point, Arbitrary Scaling Directions. Other Transformations.
- Windowing and Clipping: Windowing Concepts. Clipping Algorithms. Line Clipping, Area Clipping. Window–to–Viewport Transformation.
- Three–Dimensional Transformations: Translation, Scaling, Rotation. Rotation about an arbritary axis. Transformation Matrices. Other Transformations: Reflections, Shears.
- Projections: Perspective Projections. Parallel Projections.
- Representation of Curves: Interpolation methods. Langrange Intepolation and Cubic Splines. Approximation methods. Bezier Curves and B–Splines.

518. Algorithm Design and Analysis

- The notion of algorithm: run–time computation, proofs of correctness, recurrences, worst–case performance, average case performance.
- General methods of algorithmic design: divide and conquer method, dynamic programming, greedy algorithms.
- Graphs and graph algorithms: representation of graphs, graph transversals, minimum–cost spanning trees, shortest paths.
- Algorithms for network problems: network flows, augmenting paths, matchings in bipartite graphs, minimum cost flows.
- Topics: pattern matching, data compression, public key cryptography, approximate algorithms.

617. Computational Science and Engineering

- Introduction to scientific computing with applications to science and engineering.

618. Computational Complexity

- Models of computability, Turing machines, the notion of complexity of a problem. Complexity classes: Class PSPACE, Savitch’s theorem, Classes P and EXP.
- Non–Deterministic Turing machines.
- Classes NP and co–NP. The Projection Theorem. Reducibilities and Completeness, NP–hardness.
- Coo$k$-Levin theorem, NP–complete problems, Methods of proof of NP–completeness, Pseudo-polynomiality, Strongly NP–complete problems.
- NP–completness and approximability, EXP–complete and PSPACE–complete problems.

653. Numerical Analysis II

The course is an introduction to numerical methods for ordinary and partial differential equations. Specifically, the following topics are covered:

- Numerical solution of initial–value problems for ordinary differential equations. Euler□s method, Runge–Kutta and multistep methods. Accuracy, stability, adaptive step control. Stiff systems and absolute stability.
- Numerical solution of two–point boundary value problems for second–order ordinary differential equations with difference methods.
- Introduction to the numerical solution of partial differential equations: Finite difference methods for Laplace’□s equation, the heat equation, and the wave equation.

There is a Matlab–based computing lab as part of the course.

658. Methods of Applied Mathematics

- An Introduction to Boundary Value Problems for Second Order Ordinary Differential Equations – Sturm–Liouville Problems.
- Dimensional Analysis and Scaling.
- Asymptotic Analysis and Perturbation Methods.
- An Introduction to the Calculus of Variations.
- Integral Equations and Green’s Functions.
- An Introduction to the Partial Differential Equations of Continuum Mechanics and Wave Theory.

734. Algebraic Combinatorics

Algebraic Combinatorics is the area of mathematics which either uses tools from algebra or related fields of theoretical mathematics to solve purely combinatorial problems, or uses combinatorial methods to solve problems that come up in these fields. The objective of this course is to demonstrate this interaction through specific examples of problems, assuming only a minimum prerequisite on algebra and combinatorics. More specifically, the objective is to familiarize students in theoretical mathematics with combinatorial methods and their importance in pure mathematics students in applied mathematics with ways in which tools from pure mathematics (e.g. linear algebra) can be used to solve practical combinatorial problems. Selection from the following topics:

- Review of fundamental principles and techniques of enumeration, with an emphasis on bijective proofs and the method of generating functions. Examples (sets, permutations, integer partitions etc) (2 weeks).
- Permutations as words, elements of the symmetric group, disjoint union of cycles (cycle structure), 0–1 matrices, increasing trees etc. Permutation enumeration (inversions, cycles, descents, excedences, fixed points, alternating permutations, major index and MacMahon's Theorem). Permutations of multisets, inversions and q–binomial coefficients. Young tableaux and the hoo$k$-length formula, the Robinson–Schensted correspondence, Knuth equivalence, Schutzenberger's teasing game, applications to monotone subsequences, the evacuation tableau and Schutzenberger's theorem on the inverse and reverse permutation. The weak Bruhat order and applications on the enumeration of reduced decompositions of permutations (7 weeks).
- Elements of algebraic graph theory, the adjacency matrix of a graph (directed or not), eigenvalues and enumeration of walks. The Laplacian matrix, spanning trees and the Matrix–Tree Theorem, applications to complete (Cayley's formula) and bipartite graphs. Walks in the Young lattice and differential partially ordered sets. Applications of linear algebra on topics such as: the unimodality of q–binomial coefficients, existence of matchings in graphs and Sperner's Theorem and its generalizations (4 weeks).

739. Discrete Dynamical Systems and Applications

- Difference Calculus. First–order difference equations.
- Linear difference equations. Linear difference equations with constant coefficients. Linear partial difference equations.
- Non–linear difference equations.
- Applications from Biology, Economics, Sociology, Physics and Control Theory.

752. Numerical Linear Algebra

- Computer arithmetic. Fixed point computations, floating point computations, rounding errors in computations, numerically effective algorithms.
- Error analysis. Laws of floating point arithmetic, Addition, multiplication and inner product of n–floating point numbers, floating point matrix operations, stability of algorithms and conditioning of problems.

- Gaussian elimination and LU factorisation. LU factorisation using Gaussian elimination, partial and complete pivoting, Gauss–Jordan transformations, computation of the inverse of a matrix, Stability of Gaussian elimination. Cholesky factorisation.
- Numerical Solutions of Linear Systems. Direct methods: Solution of upper and lower triangular linear systems, solution of a system using LU factorisation and their stability, solving linear systems with multiple right–hand side. Sensitivity analysis of linear systems. Iterative methods: sparse matrices, stationary methods, conjugate gradients, preconditioning.

814. Control Theory

- Mathematical models of physical systems.
- Linearization and transfer functions, state–space approach to linear systems theory.
- Segre–Weyr method for the Jordan form of a linear operator.
- Functions of square matrices. Functions $1(t)$, (t), Laplace transform.
- General solution of time dependent linear dynamical systems.
- Root locus method.
- Controllability and Observability.
- Realisation theory. Feedback.
- Stability (general theory). Liapunov theorems.
- Stability criteria for linear dynamical systems.

815. Optimization

- Convex sets, supporting and separating planes.
- Separation theorems.
- Extreme points, Minkowski theorem.
- Polyhedra, characterization of the extreme points of polyhedra.
- Applications to Linear Programming.
- Convex functions, differentiability, maxima and minima.
- Optimization under equality and inequality constraints.
- Lagrange multiplier functions. Karush–Kuch–Tucker conditions.
- Duality, Lagrange duality and saddle points.
- Applications to Economic theory.

817. Applied Fourier Analysis

- Basic facts from the theory of Fourier series.

- Fourier Transform, Discrete Fourier Transform, Fast Fourier Transform.
- Applications to differential equations and signal processing.

Statistics and Operations Research

553. Actuarial Science

- Brief probability background with focus in compound distributions and moment generating functions.
- Compound interest.
- Annuities with fixed or random interest rate.
- Survival distributions.
- Risk Theory and, in particular, utility theory and premium calculations.
- Collective and individual risk models.
- Probability of Ruin.

555. Bayesian Inference

- Introduction to Bayesian Inference (Bayes' Theorem for evaluating the posterior distribution of an unknown parameter and applications)
- Choice of prior distributions (conjugate, non–informative, Jeffrey's priors)
- Multi–parameter problems (joint, marginal and conditional posterior distributions)
- Decision Theory (loss functions and Bayesian point estimation)
- Credibility regions and Bayesian hypothesis testing
- Bayesian model comparison
- Prediction (Bayesian predictive distributions)
- Application to changepoint models
- Applications to linear regression

Matlab and/or the statistical package R will be used for applications.

559. Game Theory

- Games in extensive form (tree representation, information sets, the notion of strategy and strategic equilibrium, Zermelo–Kuhn theorem, solution by backwards induction to find sub-game perfect equilibria).
- Games in normal form (the mixed extension of a game, normal form and the transformation of a game from extensive to normal form, Nash's Theorem).

- Matrix Games (security level of the players in pure and mixed strategies, Minimax Theorem, solution via linear programming, strategy domination, symmetric matrix games, equalizing strategies and searching of a solution via equalization, games against nature).
- Bi–Matrix games (best response correspondences, graphical solution for 2x2 games).
- Cooperative game theory (games defined by a characteristic function, axioms, examples, the von–Neumann characteristic function obtained from the normal form of a game, 0–1 normalization, classes of equivalence, characterization of dummy players, essential games and essential coalitions, the set of imputations and the core, graphical solution for the core for games with 2 or 3 players, the core in special classes of games (e.g. voting systems), the Shapley value (existence and uniqueness), closed form solution for economic and political games).

659. Linear and Non–Linear Programming

- Introduction: Convex sets and hyperplanes, separation theorems in Euclidean spaces.
- Linear Programming, the geometric picture: Basic feasible solutions and correspondence with the extreme points of the feasible solutions set, theorems pertaining to the optimal feasible solutions.
- The Simplex method, theory and tableau.
- Convex functions, continuity and differentiability of convex functions, minima of convex functions on a convex set, convex programming.
- Optimization without constraints: First and second order necessary and sufficient conditions.
- Optimization under inequality constraints: Geometric optimality conditions, Fritz John conditions, Karush–Kuhn–Tucker conditions (first order necessary conditions, geometric interpretation, first order approximation via linear programming, first order sufficient conditions).
- Optimization under both equality and inequality constraints: Geometric necessary and sufficient conditions, Fritz John and Karush–Kuhn–Tucker necessary and sufficient first order conditions.
- Applications in geometric, finance, statistical and OR problems.

669. Algorithmic Operations Research

- Mathematical models of complex operations research problems.
- Computational methods for solving mathematical programming problems.
- Computational methods for stochastic processes with large state space.
- Large production and inventory control models.

753. Multivariate Data Analysis

- Descriptive Statistics for Multidimensional Data
- Estimation for the Multivariate Normal Distribution

- Principal Components Analysis
- Factor Analysis
- Discriminant Analysis

Note: The Course includes lab sessions using one or more statistical packages/programs (SPSS, STATGRAPHICS, MATLAB).

754. Dynamic Programming
Deterministic Dynamic Programming:

- Shortest path problems in networks.
- Inventory control problems.
- Scheduling problems.
- Problems of allocation of resources, the Knapsack problem etc.

Stochastic Dynamic Programming with finite state and action spaces and finite horizon:

- Stochastic networks and path minimization.
- Myopic policies and sufficient conditions for their optimality.
- Inventory control with stochastic demand.
- Maintenance–Replacement of equipment in stochastic environment.
- Other applications.

Computational Techniques:

- Successive approximations of the value function.
- Policy improvement.
- Linear programming.

Discounted Dynamic Programming:

- Proofs of the existence of an optimal stationary policy and of the optimality equations.
- Algorithms: Successive approximations (value iteration), policy improvement, linear programming.

755. Computational Statistics

- Introduction to Simulation
- Expectation-Maximization (EM) algorithm
- Newton-Raphson algorithm
- Bootstrap
- Computer implementation (Programming) of Computational Statistics methods

854. Reliability Theory

- The role of Statistical Quality Control in production and its applications.
- Producer and Client Risk. Operating Characteristic Curve.
- Sampling with categorical and continuous variables.
- Single, double, multiple, sequential plans.
- Upper and Lower Control Limits.
- Quality Control Charts for continuous and categorical variables.
- The Structure of a system.
- Reliability as a function of time.
- Lifetime distributions.
- Aging Classes of lifetime distributions and their properties.
- Statistical Reliability Theory.

856. Stochastic Calculus

- Conditional expectation.
- Discrete time martingales.
- Continuous time martingales.
- Construction of Brownian motion, analytical properties, and related martingales.
- Stochastic integration with respect to Brownian motion.
- Ito's formula, and applications to the solution of stochastic differential equations.
- Pricing of European options, and the Blac$k$-Scholes equation.

859. Queueing Theory

- Description of queueing models: Basic notions and general results.
- Birth–death queues.
- Markovian queues and the method of phases.
- The $M|G|1$ queue and its modifications.
- The $Gl|M|k$ queue.
- Random walks and the $Gl|G|1$ queue.
- Applications.

857. Non-parameteric Statistics

- Nonparametric Tests (χ^2, Kolmogorov-Smirnov, Sign, Wilcoxon,$\ldots$)
- Density estimation
- Bootstrap theory
- Nonparametric Regression

0.6 Courses on Mathematical Education

691. Mathematics education I

- Constructivism and mathematics education: Basic principles of constructivism, The notion of scheme, The theory of conceptual fields (Vergnaud).
- Socio-cultural perspectives in mathematics education: Vygotsky's theory (thought and language, mediation, internalization, Zone of Proximal Development - ZPD).
- The notion of mathematical activity: What is a mathematical activity, Inquiry-based activities in mathematics, Basic principles of designing mathematical activities, Contextual tasks in mathematics (Realistic Mathematics education), Mathematical modeling.
- The Theory of didactical Situations (TDS): Basic elements of TDS, A-didactical situations, Didactical Engineering, TDS and design of tasks for mathematics.
- The process – object duality of mathematical concepts.
- The notion of didactic contract (DC): Rules, Ruptures, Kinds of DCs, Influences of the DC in teaching, Examples.
- Teaching and learning of algebra: The nature of algebra, Algebra in the curriculum, Conceptual and didactical aspects of the notion of function, Algebraic symbolism.
- Teaching and learning of geometry: Geometrical figures and geometrical reasoning, Cognitive processes and geometrical figures, Types of geometrical figure apprehension.

692. Digital Technologies in Mathematics Education

- Theoretical frameworks related to the use of digital technologies in mathematics education.
- Expressive digital tools in mathematics education: Symbolic expression by means of programming, Dynamic Geometry Systems, Digital tools for algebra.
- Task design based on the use of digital tools: Basic principles, Inquiry-based tasks, Links between digital representations and mathematical concepts, Dynamic manipulation of mathematical objects, Students' learning trajectories.
- Teaching and learning of geometry with the use of digital tools: Geometrical figures/constructions, Ratio and proportion, Formulating conjectures, Proof and proving.
- Teaching and learning of algebra with the use of digital tools: The notion of variable, Functional relationships, Function as covariation.

694. Historical Development of Calculus

- Eudoxus and the method of exhaustion. Archimedes: area and volume computations. The mechanical method.
- Medieval speculations on motion and variability. The analytic art of Viète. The analytic geometry of Descartes and Fermat.
- Early indivisibles and infinitesimal techniques: Kepler, Cavalieri. Arithmetical quadratures. Integration of fractional powers.

- Early tangent constructions. The methods of Fermat, Descartes, Roberval, Torricelli.
- The discovery of the binomial series. Wallis' interpolation scheme and infinite product. Newton and the binomial series.
- Logarithms: Napier's definition. Logarithms and hyperbolic integrald. Mercator's series.
- The Calculus according to Newton and Leibniz.
- 18th Century. Euler: the concept of a function, exponential and logarithmic functions, trigonometric functions and their expansion. From numerical integration to Taylor's theorem. Berkeley's criticism. Lagrange: theory of analytic functions.
- 19th Century. Fourier series. Bolzano, Cauchy and continuity. Cauchy's differential and integral calculus. Riemann's integral and its reformulations.
- Construction of the real numbers: Dedekind και Cantor.

792. Mathematics Education II.

- Mathematics Education as a scientific discipline.
- Curricula and textbooks.
- The meaning of mathematical activity.
- Exploring students' thinking in specific mathematical areas: the teaching and learning of Algebra, Geometry and Statistics at the secondary educational level.
- Teaching mathematics through problem solving.
- Argumentation and proof in mathematics teaching.
- The development of teaching materials.
- Mathematics teaching and classroom interaction.
- Social dimensions in the learning and teaching of Mathematics.

496. Ancient GreekMathematics-Euclid's Elements

In the present course the intention is the study Euclid's Elements (to a large extent from the original), the reconstruction of the history of ancient Greek Mathematics (mostly till Euclid's era) based on ancient sources and modern interpretations, the correlation with the ancient philosophy of the Pythagoreans, Eleatics, and Plato, and the relation of ancient Greek Mathematics with modern Mathematics (natural numbers, rational numbers and mathematical induction, real numbers, and Calculus).

- The least number principle and mathematical induction, Euclidean algorithm, and the greatest common divisor of two numbers, the theory of ratios of numbers and its relation to rational numbers, the fundamental theorem of Arithmetic (Book 7 of the Elements), the infinity of prime numbers. The origin of the theory of ratios of numbers from Pythagorean music (Philolaos).
- The axiomatic foundation of Geometry. The first half of Book 1 without the Fifth Postulate, Pythagorean theorem, application of areas and Geometric Algebra, incommensurable magnitudes (Hippasus), infinite anthyphairesis, sid and diameter numbers(Books 1 and 2 of the Elements).The philosophy of the Pythagoreans and Zeno's paradoxes. Hippocrates from Chios and quadrature of the lunules.

- Incommesurabilities (Theodorus, Theaetetus, Archytas). The theory of ratios of magnitudes: the anthyphairetic Theaetetus theory and its relation with Plato’s philosophy, Eudoxus theory (Books 5 and 6 of the Elements) and its relation to the modern foundation of the real numbers with Dedekind cuta. Application of Eudoxus theory to the method of Exhaustion (Boοκ 12 of the Elements by Eudoxus, and the work of Archimedes) and its relation to modern integral and infinitesimal calculus.

573. History of Mathematics: From Antiquity to the Renaissance

- Mesopotamian and Egyptian Mathematics Arithmetical systems and operations. The “rule of the hypotenuse” in Babylonian tablets. The so-called “Babylonian algebra” and the historiographical disputes about it. Problem-solving via numerical procedures.
- Ancient Greek Mathematics (I): Pre-Euclidean Mathematics Arithmetical systems and logistic. The creation of postulatory deductive mathematics. The famous problems of Greek geometry. Pythagorean arithmetic. Incommensurability. The so-called “geometrical algebra” and the historiographical disputes about it.
- Ancient Greek Mathematics (II): Archimedes and Apollonius The quadratures and cubatures of Archimedes. Heuristics and proofs in Archimedes. The Palimpsest of Archimedes. The Conics of Apollonius.
- Ancient Greek Mathematics (III): Late Antiquity The commentators of the Late Antiquity. Diophantus and problem-solving with algebra. Historiographical disputes about the place of Diophantus in the history of algebra.
- Mathematics in the Middle-Ages Mathematics in the Islamic World. Historiographical disputes concerning the role of Islam in the history of mathematics. Mathematics in Medieval Europe. The role of Byzantium in the history of mathematics.
- Mathematics in the Renaissance and the Early Modern Period Renaissance Algebra: The solution of 3rd and 4th degree equations. The work of François Viète. The invention of analytic geometry: Pierre Fermat and René Descartes. The precursors of infinitesimal calculus.

613. Philosophy of Mathematics.

- The ontological status of mathematical objects
- The informative content of mathematical propositions
- Mathematical truth in the context of the interpretation/structure of mathematical language
- Mathematical description of empirical reality
- The continuum, empirical reality and the problem of exact measurement
- The notion of the infinite and the notion of the continuum according to Aristotle
- The notion of the continuum according to Leibniz
- Time and the continuum
- Mathematical facts, mathematical events
- Hume’s skepticism about empirical induction and the contemporary debate

- Frege's attempt of arithmetic's reduction to logic and the contemporary version of logicism
- Ontology of numbers (Aristotle, Mill, Frege)
- The problem of mathematical knowledge: rationalism and empirical accounts
- Constructivism in mathematics – intuitionism

897. Epistemology and Mathematics teaching

- Introduction: the basic epistemological problems about possibility and validity of scientific knowledge.
- Epistemological problems in antiquity (Plato-Aristotle). The conceptions of dialectic knowledge and proof.
- Descartes' rationalism. Innate mathematical patterns and deductive proof.
- English empiricism (Locke, Hume). The empirical origin of knowledge. Empiricism about mathematical notions.
- The basic elements of Kant's account of mathematics. Synthetic a priori knowledge of arithmetic and geometry.
- The problem of foundations of mathematics in 19th-20th century and the contestation of intuition. The philosophical programs of Fregean Logicism, Brouwer's Intuitionism and Hilbert's Formalism. Dealing with paradoxes.
- The problem of the combination between standard semantics of mathematical language and mathematical knowledge (Benacerraf's dilemma).
- 20th cent. epistemological approaches of Popper, Kuhn and Lakatos and their impact on epistemological problems concerning mathematics.
- Learning as a constructive theory (Von Glasersfeld)
- Socio-cultural perspectives of Mathematics teaching (Vygotsky, Leontiev, Wenger). Activity Theory. Ethnomathematics.
- Theory of objectification (Radford).
- Embodied knowledge (Lakoff).
- Anthropological Approaches in Mathematics Education, French Perspectives.
- Epistemological obstacles (Bachelard, Brousseau)
- How mathematics teachers' epistemological beliefs could affect their teaching practices.

693. Geometry Education

The course aims the students: to understand the role of geometry in mathematics education; improve their knowledge of school geometry; to develop their knowledge about how pupils think related to geometry; to be familiarized with new teaching approaches. The basic content areas are:

- Historical development of geometry and basic epistemological issues.

- Geometry and spatial ability.
- The development of geometrical thinking and the role of visualization.
- Teaching and learning geometry in mathematics curriculum.
- Learning and teaching basic geometrical concepts (e.g. shape, angle).
- Geometrical transformations as tools of exploring geometrical properties and justifications.
- Geometrical measures (e.g. measurement of length, area, space): Basic processes and the role of measuring tools.
- Proof in geometry, students' proof schemes and teaching approaches (e.g. structural and conceptual elements, key idea, conjecture and proof).
- Using concrete and digital tools in the teaching of geometry.

898. Teaching through problem solving – Mathematization

- Definitions, Polya & Schoenfeld (heuristic strategies, teachers' beliefs, teaching resources and metacognition).
- Educational reforms on teaching through problem solving.
- Curricular resources and teaching goals. Types of problems (e.g., context free or context specific, open ended or closed).
- Mathematization. The role of context in teaching through problem solving-Modelling. Problems in different contexts (e.g., every-day life problems, authentic workplace problems) and how we can bring them in our every-day classroom teaching.
- Teaching issues in problem solving activities: Problem posing by teachers and pupils. Teaching phases: Introducing the problem into the classroom; students' autonomous work; whole classroom discussion. Evaluating students' work.
- Special issues on teaching through problem solving: Problem solving and inquiry-based teaching; problem solving in multicultural classrooms; problem solving and differentiating instruction; problem solving and argumentation.

591. Teaching of Calculus

The aim of the course

The subject of this course is the mathematical knowledge for the teaching of calculus. This consists of the subject matter knowledge and the pedagogical content knowledge of calculus. Pedagogical content knowledge is the knowledge needed to transform the subject matter knowledge into knowledge for teaching. The students have acquired the subject matter knowledge through the courses Calculus I and Calculus II. However, research data has shown that many students fail to grasp this knowledge in a didactical environment. In the beginning of this course, general issues on the teaching of mathematics that are essential for the teaching of Calculus are being presented, and afterwards the mathematical knowledge for teaching of the basic concepts of Calculus will be discussed.

Content

- The importance of definitions in the teaching and learning of Mathematics.
- The importance of visual representations in the teaching and learning of Mathematics.
- The teaching of concepts and theorems
- General issues on the teaching of Calculus.
- The mathematical knowledge for the teaching of the limit.
- The mathematical knowledge for the teaching of the continuity
- The mathematical knowledge for the teaching of of the derivative
- The mathematical knowledge for the teaching of of the integral
- Examples of teaching approaches for some concepts and theorems of Calculus

0.7 Lessons for acquiring professional experience

795. Mathematics teaching practice in secondary schools

The course aims to help students link their knowledge of teaching and learning (provided mostly by mathematics education courses) with actual teaching in secondary schools. Every second week for the entire semester students are asked to participate in a number of field activities while each week following the activities in schools includes a three-hour meeting at the university. Students' field activities consist of observing other teachers' mathematics teaching in schools as well as designing and teaching lessons in the classroom. At the university, the students focus on different areas of mathematics included in the curriculum (e.g., algebra, geometry, functions) by a number of activities such as: presentation and discussion of research papers; observation and analysis of video-taped lessons (if this is possible); study of the curriculum and textbooks; analysis of tasks and teaching situations (e.g., focus on students' responses); presentation of critical events based on their observations and in the analysis of their own teaching; discussion and questioning of emerging issues (e.g., linking analysis of events to research findings); development of alternative teaching actions; design of teaching materials (e.g., tasks, worksheets, digital resources). Participation in the lesson is compulsory. Evaluation is based on students' portfolios and final written exams.